Edexcel Maths FP2

Topic Questions from Papers

Complex Numbers

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2. Solve the equation

$$z^3 = 4\sqrt{2} - 4\sqrt{2}\text{i},$$

giving your answers in the form $r(\cos\theta + \text{i}\sin\theta)$, where $-\pi < \theta \leqslant \pi$.

(6)

Leave blank

6. A transformation T from the z-plane to the w-plane is given by

$$w = \frac{z}{z + \mathrm{i}}, \quad z \neq -\mathrm{i}$$

The circle with equation $|z| = 3$ is mapped by T onto the curve C.

(a) Show that C is a circle and find its centre and radius.

(8)

The region $|z| < 3$ in the z-plane is mapped by T onto the region R in the w-plane.

(b) Shade the region R on an Argand diagram.

(2)

Leave blank

Question 6 continued

Leave blank

4. $$z = -8 + (8\sqrt{3})\text{i}$$

(a) Find the modulus of z and the argument of z.

(3)

Using de Moivre's theorem,

(b) find z^3,

(2)

(c) find the values of w such that $w^4 = z$, giving your answers in the form $a + \text{i}b$, where $a, b \in \mathbb{R}$.

(5)

Leave blank

6. A complex number z is represented by the point P in the Argand diagram.

(a) Given that $|z-6|=|z|$, sketch the locus of P.

(2)

(b) Find the complex numbers z which satisfy both $|z-6|=|z|$ and $|z-3-4\mathrm{i}|=5$.

(3)

The transformation T from the z-plane to the w-plane is given by $w=\dfrac{30}{z}$.

(c) Show that T maps $|z-6|=|z|$ onto a circle in the w-plane and give the cartesian equation of this circle.

(5)

Leave blank

Question 6 continued

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5. The point P represents the complex number z on an Argand diagram, where

$$|z - \mathrm{i}| = 2$$

The locus of P as z varies is the curve C.

(a) Find a cartesian equation of C.

(2)

(b) Sketch the curve C.

(2)

A transformation T from the z-plane to the w-plane is given by

$$w = \frac{z + \mathrm{i}}{3 + \mathrm{i}z}, \quad z \neq 3\mathrm{i}$$

The point Q is mapped by T onto the point R. Given that R lies on the real axis,

(c) show that Q lies on C.

(5)

Leave blank

Question 5 continued

Leave blank

7. (a) Use de Moivre's theorem to show that

$$\sin 5\theta = 16\sin^5\theta - 20\sin^3\theta + 5\sin\theta$$

(5)

Hence, given also that $\sin 3\theta = 3\sin\theta - 4\sin^3\theta$,

(b) find all the solutions of

$$\sin 5\theta = 5\sin 3\theta,$$

in the interval $0 \leqslant \theta < 2\pi$. Give your answers to 3 decimal places.

(6)

Leave blank

Question 7 continued

Leave blank

3. (a) Express the complex number $-2+\left(2\sqrt{3}\right)\mathrm{i}$ in the form $r(\cos\theta+\mathrm{i}\sin\theta),\ -\pi<\theta\leqslant\pi$.
(3)

(b) Solve the equation

$$z^4=-2+\left(2\sqrt{3}\right)\mathrm{i}$$

giving the roots in the form $r(\cos\theta+\mathrm{i}\sin\theta),\ -\pi<\theta\leqslant\pi$.
(5)

Leave blank

8. The point P represents a complex number z on an Argand diagram such that

$$|z - 6\text{i}| = 2|z - 3|$$

(a) Show that, as z varies, the locus of P is a circle, stating the radius and the coordinates of the centre of this circle.

(6)

The point Q represents a complex number z on an Argand diagram such that

$$\arg(z - 6) = -\frac{3\pi}{4}$$

(b) Sketch, on the same Argand diagram, the locus of P and the locus of Q as z varies.

(4)

(c) Find the complex number for which both $|z - 6\text{i}| = 2|z - 3|$ and $\arg(z - 6) = -\dfrac{3\pi}{4}$

(4)

Leave blank

Question 8 continued

Q8

(Total 14 marks)

TOTAL FOR PAPER: 75 MARKS

END

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1. A transformation T from the z-plane to the w-plane is given by

$$w = \frac{z + 2\text{i}}{\text{i}z} \qquad z \neq 0$$

The transformation maps points on the real axis in the z-plane onto a line in the w-plane.

Find an equation of this line.

(4)

Leave blank

6. The complex number $z = \mathrm{e}^{\mathrm{i}\theta}$, where θ is real.

(a) Use de Moivre's theorem to show that

$$z^n + \frac{1}{z^n} = 2\cos n\theta$$

where n is a positive integer.

(2)

(b) Show that

$$\cos^5\theta = \frac{1}{16}(\cos 5\theta + 5\cos 3\theta + 10\cos\theta)$$

(5)

(c) Hence find all the solutions of

$$\cos 5\theta + 5\cos 3\theta + 12\cos\theta = 0$$

in the interval $0 \leqslant \theta < 2\pi$

(4)

Leave blank

Question 6 continued

Leave blank

2. $$z = 5\sqrt{3} - 5\text{i}$$

Find

(a) $|z|$, **(1)**

(b) $\arg(z)$, in terms of π. **(2)**

$$w = 2\left(\cos\frac{\pi}{4} + \text{i}\sin\frac{\pi}{4}\right)$$

Find

(c) $\left|\frac{w}{z}\right|$, **(1)**

(d) $\arg\left(\frac{w}{z}\right)$, in terms of π. **(2)**

Leave blank

4. (a) Given that

$$z = r(\cos\theta + \mathrm{i}\sin\theta), \qquad r \in \mathbb{R}$$

prove, by induction, that $z^n = r^n(\cos n\theta + \mathrm{i}\sin n\theta), \qquad n \in \mathbb{Z}^+$

(5)

$$w = 3\left(\cos\frac{3\pi}{4} + \mathrm{i}\sin\frac{3\pi}{4}\right)$$

(b) Find the exact value of w^5, giving your answer in the form $a + \mathrm{i}b$, where $a, b \in \mathbb{R}$.

(2)

Leave blank

Question 4 continued

Further Pure Mathematics FP2

Candidates sitting FP2 may also require those formulae listed under Further Pure Mathematics FP1 and Core Mathematics C1–C4.

Area of a sector

$$A = \frac{1}{2}\int r^2 \, \mathrm{d}\theta \qquad \text{(polar coordinates)}$$

Complex numbers

$$\mathrm{e}^{\mathrm{i}\theta} = \cos\theta + \mathrm{i}\sin\theta$$

$$\{r(\cos\theta + \mathrm{i}\sin\theta)\}^n = r^n(\cos n\theta + \mathrm{i}\sin n\theta)$$

The roots of $z^n = 1$ are given by $z = \mathrm{e}^{\frac{2\pi k\mathrm{i}}{n}}$, for $k = 0, 1, 2, \ldots, n-1$

Maclaurin's and Taylor's Series

$$\mathrm{f}(x) = \mathrm{f}(0) + x\,\mathrm{f}'(0) + \frac{x^2}{2!}\mathrm{f}''(0) + \ldots + \frac{x^r}{r!}\mathrm{f}^{(r)}(0) + \ldots$$

$$\mathrm{f}(x) = \mathrm{f}(a) + (x-a)\,\mathrm{f}'(a) + \frac{(x-a)^2}{2!}\mathrm{f}''(a) + \ldots + \frac{(x-a)^r}{r!}\mathrm{f}^{(r)}(a) + \ldots$$

$$\mathrm{f}(a+x) = \mathrm{f}(a) + x\,\mathrm{f}'(a) + \frac{x^2}{2!}\mathrm{f}''(a) + \ldots + \frac{x^r}{r!}\mathrm{f}^{(r)}(a) + \ldots$$

$$\mathrm{e}^x = \exp(x) = 1 + x + \frac{x^2}{2!} + \ldots + \frac{x^r}{r!} + \ldots \qquad \text{for all } x$$

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \ldots + (-1)^{r+1}\frac{x^r}{r} + \ldots \qquad (-1 < x \leq 1)$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \ldots + (-1)^r \frac{x^{2r+1}}{(2r+1)!} + \ldots \qquad \text{for all } x$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \ldots + (-1)^r \frac{x^{2r}}{(2r)!} + \ldots \qquad \text{for all } x$$

$$\arctan x = x - \frac{x^3}{3} + \frac{x^5}{5} - \ldots + (-1)^r \frac{x^{2r+1}}{2r+1} + \ldots \qquad (-1 \leq x \leq 1)$$

Further Pure Mathematics FP1

Candidates sitting FP1 may also require those formulae listed under Core Mathematics C1 and C2.

Summations

$$\sum_{r=1}^{n} r^2 = \tfrac{1}{6}n(n+1)(2n+1)$$

$$\sum_{r=1}^{n} r^3 = \tfrac{1}{4}n^2(n+1)^2$$

Numerical solution of equations

The Newton-Raphson iteration for solving $\mathrm{f}(x) = 0$: $x_{n+1} = x_n - \dfrac{\mathrm{f}(x_n)}{\mathrm{f}'(x_n)}$

Conics

	Parabola	**Rectangular Hyperbola**
Standard Form	$y^2 = 4ax$	$xy = c^2$
Parametric Form	$(at^2, 2at)$	$\left(ct, \dfrac{c}{t}\right)$
Foci	$(a, 0)$	Not required
Directrices	$x = -a$	Not required

Matrix transformations

Anticlockwise rotation through θ about O: $\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$

Reflection in the line $y = (\tan\theta)x$: $\begin{pmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta \end{pmatrix}$

In FP1, θ will be a multiple of 45°.

Core Mathematics C4

Candidates sitting C4 may also require those formulae listed under Core Mathematics C1, C2 and C3.

Integration **(+ *constant*)**

$\mathbf{f}(x)$	$\int \mathbf{f}(x)\ \mathrm{d}x$
$\sec^2 kx$	$\frac{1}{k}\tan kx$
$\tan x$	$\ln\lvert\sec x\rvert$
$\cot x$	$\ln\lvert\sin x\rvert$
$\operatorname{cosec} x$	$-\ln\lvert\operatorname{cosec} x + \cot x\rvert, \quad \ln\lvert\tan(\tfrac{1}{2}x)\rvert$
$\sec x$	$\ln\lvert\sec x + \tan x\rvert, \quad \ln\lvert\tan(\tfrac{1}{2}x + \tfrac{1}{4}\pi)\rvert$

$$\int u\frac{\mathrm{d}v}{\mathrm{d}x}\,\mathrm{d}x = uv - \int v\frac{\mathrm{d}u}{\mathrm{d}x}\,\mathrm{d}x$$

Core Mathematics C3

Candidates sitting C3 may also require those formulae listed under Core Mathematics C1 and C2.

Logarithms and exponentials

$$e^{x \ln a} = a^x$$

Trigonometric identities

$$\sin(A \pm B) \equiv \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) \equiv \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) \equiv \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B} \qquad (A \pm B \neq (k + \tfrac{1}{2})\pi)$$

$$\sin A + \sin B \equiv 2 \sin \frac{A+B}{2} \cos \frac{A-B}{2}$$

$$\sin A - \sin B \equiv 2 \cos \frac{A+B}{2} \sin \frac{A-B}{2}$$

$$\cos A + \cos B \equiv 2 \cos \frac{A+B}{2} \cos \frac{A-B}{2}$$

$$\cos A - \cos B \equiv -2 \sin \frac{A+B}{2} \sin \frac{A-B}{2}$$

Differentiation

f(x)	**f′(x)**
$\tan kx$	$k \sec^2 kx$
$\sec x$	$\sec x \tan x$
$\cot x$	$-\mathrm{cosec}^2\, x$
$\mathrm{cosec}\, x$	$-\mathrm{cosec}\, x \cot x$
$\dfrac{\mathrm{f}(x)}{\mathrm{g}(x)}$	$\dfrac{\mathrm{f}'(x)\mathrm{g}(x) - \mathrm{f}(x)\mathrm{g}'(x)}{(\mathrm{g}(x))^2}$

Core Mathematics C2

Candidates sitting C2 may also require those formulae listed under Core Mathematics C1.

Cosine rule

$$a^2 = b^2 + c^2 - 2bc \cos A$$

Binomial series

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1}b + \binom{n}{2} a^{n-2}b^2 + \ldots + \binom{n}{r} a^{n-r}b^r + \ldots + b^n \qquad (n \in \mathbb{N})$$

$$\text{where } \binom{n}{r} = {}^n\mathrm{C}_r = \frac{n!}{r!(n-r)!}$$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{1\times 2}x^2 + \ldots + \frac{n(n-1)\ldots(n-r+1)}{1\times 2\times \ldots \times r}x^r + \ldots \qquad (|x| < 1, n \in \mathbb{R})$$

Logarithms and exponentials

$$\log_a x = \frac{\log_b x}{\log_b a}$$

Geometric series

$$u_n = ar^{n-1}$$

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$S_\infty = \frac{a}{1-r} \text{ for } |r| < 1$$

Numerical integration

The trapezium rule: $\int_a^b y \, dx \approx \frac{1}{2}h\{(y_0 + y_n) + 2(y_1 + y_2 + ... + y_{n-1})\}$, where $h = \frac{b-a}{n}$

Core Mathematics C1

Mensuration

Surface area of sphere = $4\pi r^2$

Area of curved surface of cone = $\pi r \times$ slant height

Arithmetic series

$u_n = a + (n - 1)d$

$S_n = \frac{1}{2}n(a + l) = \frac{1}{2}n[2a + (n - 1)d]$